Tripartite Entanglement-Dependence of Tripartite Non-locality in Non-inertial Frame

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Abstract

The three-tangle-dependence of $S_{max} = \max\langle S \rangle$, where S is Svetlichny operator, are explicitly derived when one party moves with an uniform acceleration with respect to other parties in the generalized Greenberger-Horne-Zeilinger and maximally slice states. The π-tangle-dependence of S_{max} are also derived implicitly. The physical implications of quantum mechanical non-locality inferred from these dependence are briefly discussed.

After Einstein-Podolsky-Rosen's seminal paper[1] the unusual properties of the quantum correlations became a fundamental issue in quantum information theories. This unusual properties become manifest if one examines Bell inequality $\langle \mathcal{B} \rangle \leq 2$ [2] by making use of bipartite quantum states. If this inequality is violated. this fact guarantees the non-locality of quantum mechanics. As Gisin[3] showed, the Bell-type Clauser-Horner-Shimony-Holt (CHSH)[4] inequality is violated for all pure entangled two-qubit states. This fact implies that quantum mechanics really exhibits non-local correlations. More importantly, the amount of violation $\langle \mathcal{B} \rangle - 2$ increases when the two-qubit state is entangled more and more. This fact implies that the origin of the non-local correlations in quantum mechanics is an entanglement of quantum states. This remarkable fact can be used to implement the quantum cryptography[5].

Although the relationship between non-locality and entanglement is manifest to a great extent in two-qubit system, it is not straightforward to explore this relationship in multi-partite system. Recently, however, understanding in this direction is enhanced little bit, especially in three-qubit system. In Ref. [6] the relationship between Svetlichny inequality[7], the Bell-type inequality in tripartite system, and tripartite residual entanglement called three-tangle [8] was examined by making use of the generalized Greenberger-Horne-Zeilinger (GHZ) states $|\psi_g\rangle$ [9] and the maximally slice (MS) states $|\psi_s\rangle$ [10] defined as

$$\begin{aligned}
&|\psi_g\rangle = \cos\theta_1 |000\rangle + \sin\theta_1 |111\rangle \\
&|\psi_s\rangle = \frac{1}{\sqrt{2}} \bigg[|000\rangle + |11\rangle \Big\{ \cos\theta_3 |0\rangle + \sin\theta_3 |1\rangle \Big\} \bigg].
\end{aligned} \tag{1}$$

The most remarkable fact Re.[6] found is that the τ(three-tangle)-dependence of S_{max}, the upper bound of expectation value of the Svetlichny operator, for $|\psi_g\rangle$ is

$$S_{max}(\psi_g) = \begin{cases} 4\sqrt{1-\tau} & \tau \leq 1/3 \\ 4\sqrt{2\tau} & \tau \geq 1/3. \end{cases} \tag{2}$$

Since the Svetlichny inequality is $\langle S \rangle \leq 4$, whose violation guarantees the non-local correlations, Eq. (2) shows that $|\psi_g\rangle$ really exhibits non-local correlations in the region $\tau \geq 1/2$. Unlike two-qubit states, however, S_{max} exhibits a decreasing behavior when $\tau \leq 1/3$. This fact strongly suggests that the quantum entanglement is not the only resource for the multipartite non-locality. It seems to be greatly important issue to find the other resources, which are responsible for the non-local property of quantum mechanics.

The purpose of this paper is to examine the relationship between tripartite entanglement and S_{max} in non-inertial frame. Although similar issue was considered recently in Ref.[11], authors in this reference chose only π-tangle [12] as a tripartite entanglement measure. As far as we know, however, there are two different tripartite entanglement measures such as three-tangle[8] and π-tangle[12]. Unlike π-tangle the three-tangle has its own historical background. In fact, it exactly coincides with the modulus of a Cayley's hyperdeterminant[13, 14], which was constructed long ago. It is also polynomial invariant under the local $SL(2, \mathbb{C})$ transformation[15, 16].

Moreover, the calculation of three-tangle for three-qubit mixed states is much more difficult than that of π-tangle. Since three-tangle for mixed state ρ is defined by convex roof method[17, 18]

$$\tau(\rho) = \min \sum_j P_j \tau(\rho_j), \tag{3}$$

where minimum is taken over all possible ensembles of pure states ρ_j with $0 \leq P_j \leq 1$, the explicit computation of three-tangle needs to derive an optimal decomposition of the given mixed state ρ. It causes difficulties in the analytic computation of the three-tangle. Recently, however, various techniques[19–24] were developed to overcome these difficulties. In this paper we use these techniques to derive the relation between the three-tangle and S_{max} in non-inertial frames.

Now, we assume that Alice, Bob, and Charlie initially share the generalized fermionic GHZ state $|\psi_g\rangle_{ABC}$. We also assume that after sharing his own qubit, Charlie moves with respect to Alice and Bob with a uniform acceleration a. Then, Charlie's vacuum and one-particle states $|0\rangle_M$ and $|1\rangle_M$, where the subscript M stands for Minkowski, are transformed into[25]

$$\begin{aligned} |0\rangle_M &\rightarrow \cos r |0\rangle_I |0\rangle_{II} + \sin r |1\rangle_I |1\rangle_{II} \\ |1\rangle_M &\rightarrow |1\rangle_I |0\rangle_{II}, \end{aligned} \tag{4}$$

where the parameter r is defined by

$$\cos r = \frac{1}{\sqrt{1 + \exp(-2\pi\omega c/a)}}, \tag{5}$$

and c is the speed of light, and ω is the central frequency of the fermion wave packet[1]. Thus, $r = 0$ when $a = 0$ and $r = \pi/4$ when $a = \infty$. In Eq. (4) $|n\rangle_I$ and $|n\rangle_{II}$ $(n = 0, 1)$ are the mode decomposition in the two causally disconnected regions in Rindler space. Therefore, Eq. (4) implies that the physical information initially formed in region I is leaked into the region II, which is a main story of the Unruh effect[26, 27].

Using Eq. (4) one can easily show that the Charlie's acceleration makes $|\psi\rangle_{ABC}$ to be

$$|\psi\rangle_{ABC} \to \Big[\cos\theta_1 \cos r|000\rangle + \sin\theta_1|111\rangle\Big] \otimes |0\rangle_{II} + \cos\theta_1 \sin r|001\rangle \otimes |1\rangle_{II}, \tag{6}$$

where $|\alpha\beta\gamma\rangle \equiv |\alpha\beta\rangle^M_{AB} \otimes |\gamma\rangle_I$. Since $|\psi\rangle_{II}$ is a physically inaccessible state from region I, it is reasonable to take a partial trace to average it out. Then, the remaining quantum state becomes the following mixed state:

$$\begin{aligned} \rho_{ABI} = &\cos^2\theta_1 \cos^2 r|000\rangle\langle 000| + \cos^2\theta_1 \sin^2 r|001\rangle\langle 001| + \sin^2\theta_1|111\rangle\langle 111| \\ &+ \sin\theta_1\cos\theta_1\cos r\Big\{|000\rangle\langle 111| + |111\rangle\langle 000|\Big\}. \end{aligned} \tag{7}$$

The maximum of the expectation value of the Svetlichny operator, S_{max}, for ρ_{ABI} was explicitly derived in Ref.[11], and the final expression can be written as

$$S_{max} = 4\max\Big[|2\cos^2\theta_1\cos^2 r - 1|, \sqrt{2}|\sin 2\theta_1|\cos r\Big]. \tag{8}$$

When $a = 0$, Eq. (8) reduces to $S_{max} = 4\max\big[|2\cos^2\theta_1 - 1|, \sqrt{2}|\sin 2\theta_1|\big]$, which ensures that the violation of the Svetlichny inequality arises when $\pi/8 < \theta_1 < 3\pi/8$ in a region $0 \leq \theta_1 \leq \pi/2$. When $a = \infty$, Eq. (8) reduces to $S_{max} = 4\max\left[1 - \cos^2\theta_1, \sin 2\theta_1\right]$, which shows that there is no violation of the Svetlichny inequality.

Now, we discuss on the tripartite entanglement of ρ_{ABI} given in Eq. (7). The computation of its π-tangle is straightforward and the final expression becomes

$$\pi_{GGHZ} = \frac{2 + \cos^2 r}{3}\sin^2 2\theta_1 + \frac{1}{3}\cos^4\theta_1 \sin^2 2r. \tag{9}$$

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[1] For bosonic state Eq.(4) is changed into

$$|0\rangle_M \to \frac{1}{\cosh r}\sum_{n=0}^{\infty} \tanh^n r|n\rangle_I|n\rangle_{II} \qquad |1\rangle_M \to \frac{1}{\cosh^2 r}\sum_{n=0}^{\infty} \tanh^n r\sqrt{n+1}|n+1\rangle_I|n\rangle_{II},$$

where

$$\cosh r = \frac{1}{\sqrt{1 - \exp(-2\pi\omega c/a)}}$$

.

When, therefore, $a = 0$, π_{GGHZ} becomes $\sin^2 2\theta_1$, which shows that $|\psi_g\rangle$ is maximally entangled at $\theta_1 = \pi/4$ and non-entangled at $\theta_1 = 0$ and $\pi/2$. When $a = \infty$, Eq. (9) reduces to $\pi_{GGHZ} = (5/6)\sin^2 2\theta_1 + (1/3)\cos^4\theta_1$, which is maximized by $25/27 \sim 0.926$ at $\theta_1 = \sin^{-1}(2/3)$ and minimized by zero at $\theta_1 = \pi/2$. The nonvanishing tripartite entanglement at $a \to \infty$ limit was discussed in Ref.[28]. This property is a crucial difference from the bosonic bipartite entanglement, which completely vanishes at $a \to \infty$ limit[29].

In order to compute the three-tangle it is convenient to use the spectral decomposition of ρ_{ABI}, whose expression is

$$\rho_{ABI} = p|GHZ\rangle\langle GHZ| + (1-p)|001\rangle\langle 001|, \tag{10}$$

where $|GHZ\rangle = a|000\rangle + b|111\rangle$ with

$$p = \cos^2\theta_1\cos^2 r + \sin^2\theta_1 \qquad a = \frac{\cos\theta_1\cos r}{\sqrt{\sin^2\theta_1 + \cos^2\theta_1\cos^2 r}} \qquad b = \frac{\sin\theta_1}{\sqrt{\sin^2\theta_1 + \cos^2\theta_1\cos^2 r}}. \tag{11}$$

In order to derive the optimal decomposition we define

$$|Z(\phi)\rangle = \sqrt{p}|GHZ\rangle + e^{i\phi}\sqrt{1-p}|001\rangle. \tag{12}$$

This has several interesting properties. First, ρ_{ABI} given in Eq.(10) can be written as

$$\rho_{ABI} = \frac{1}{2}\Big[|Z(\phi)\rangle\langle Z(\phi)| + |Z(\phi+\pi)\rangle\langle Z(\phi+\pi)|\Big]. \tag{13}$$

Second, the three-tangle of $|Z(\phi)\rangle$ is independent of ϕ as $\tau_Z = 4p^2a^2b^2$. If, therefore, Eq. (13) is an optimal decomposition, the three-tangle of ρ_{ABI} is also $\tau_{ABI} = 4p^2a^2b^2$. Since τ_{ABI} is convex with respect to p, this fact guarantees that Eq. (13) is really optimal decomposition for ρ_{ABI}. Using Eq. (11) it is easy to show

$$\tau_{ABI} = \sin^2 2\theta_1\cos^2 r. \tag{14}$$

Therefore, combining Eq. (8) and Eq. (14) we get the explicit three-tangle-dependence of S_{max} as following;

$$S_{max} = 4\max\left[\sqrt{\cos^2 r - \tau_{ABI}}\cos r - \sin^2 r, \sqrt{2\tau_{ABI}}\right]. \tag{15}$$

When $a = 0$, it is easy to to show that Eq. (2) is reproduced.

In Fig. 1 (a) we plot the three-tangle-dependence of π-tangle when $a = 0$, $2\omega c$, $5\omega c$, and $10\omega c$. As expected from a fact that these are two different tripartite entanglement measures,

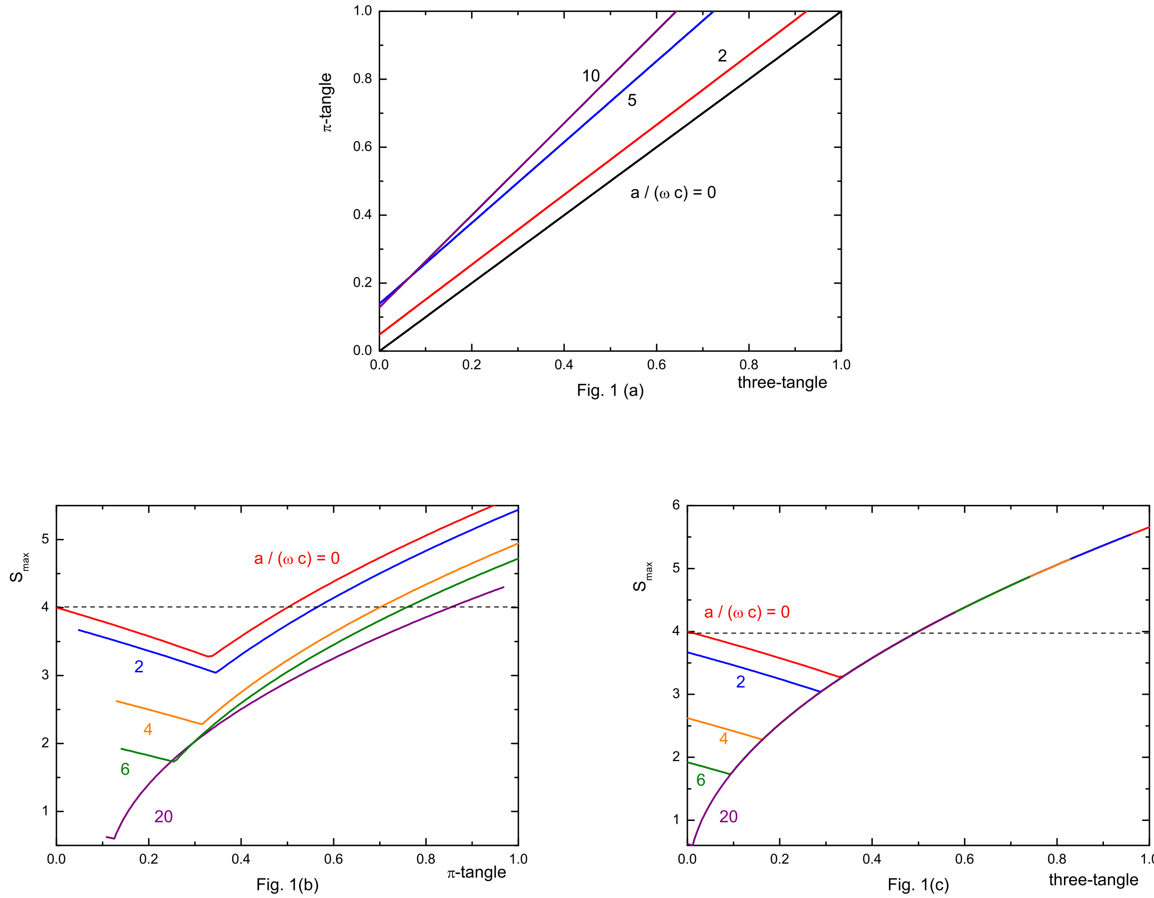

FIG. 1: (Color online) In (a) we plot the π-tangle (9) versus three-tangle (14). The π-tangle exhibits monotonous behavior with respect to the three-tangle. This fact is plausible because these tangles are two different measures for tripartite entanglement. In (b) and (c) we plot the tripartite entanglement-dependence of S_{max}. These figures show that S_{max} exhibits a decreasing behavior in the small entanglement region. This fact seems to imply that the entanglement is not unique physical resource for quantum mechanical non-locality.

π-tangle is monotonous with respect to three-tangle. Fig. 1(a) also shows that regardless of acceleration a π-tangle is larger than three-tangle, which was conjectured in Ref.[12, 23].

$a/\omega c$	0	2	4	6	8	10	100	∞
π_*	0.50	0.563	0.70	0.757	0.787	0.806	0.901	1
τ_*	1	0.959	0.828	0.740	0.687	0.652	0.566	0.5

Table I: Acceleration dependence of π_* and τ_*

Fig. 1(b) and Fig. 1(c) show the tripartite entanglement-dependence of S_{max}. As Fig. 1(b) exhibits, the violation of the Svetlichny inequality, i.e. $S_{max} > 4$, occurs when $\pi_{ABI} > \pi_*$, where π_* increases with increasing a. The critical value π_* is given in Table I for various a. As Table I shows, π_* approaches 1 at $a \to \infty$ limit, which implies that there is no violation of the Svetlichny inequality in this limit. Fig. 1(c) is a plot for the τ_{ABI}-dependence of S_{max} for various a. As Fig. 1(c) exhibits, the violation of the Svetlichny inequality occurs when $\tau_{ABI} > 0.5$ for all a. The maximum of the three-tangle, i.e. τ_*, is dependent on Charlie's acceleration a. As Table I shows, τ_* exhibits a decreasing behavior with increasing a, and eventually approaches 0.5 in $a \to \infty$ limit. This fact also indicates that the state shared initially by Alice, Bob, and Charlie cannot have non-local property in the infinite Charlie's acceleration although it has nonzero tripartite entanglement.

If Alice, Bob, and Charlie share initially the MS state $|\psi_s\rangle_{ABC}$, Charlie's acceleration changes $|\psi_s\rangle_{ABC}$ into

$$\begin{aligned}\sigma_{ABI} = \frac{1}{2}\Bigg[& \cos^2 r|000\rangle\langle 000| + \sin^2 r|001\rangle\langle 001| + \cos^2\theta_3 \cos^2 r|110\rangle\langle 110| \\ & + \left(\sin^2\theta_3 + \cos^2\theta_3 \sin^2 r\right)|111\rangle\langle 111| \\ & + \cos\theta_3 \cos^2 r\left\{|000\rangle\langle 110| + |110\rangle\langle 000|\right\} + \sin\theta_3 \cos r\left\{|000\rangle\langle 111| + |111\rangle\langle 000|\right\} \\ & + \cos\theta_3 \sin^2 r\left\{|001\rangle\langle 111| + |111\rangle\langle 001|\right\} + \sin\theta_3\cos\theta_3\cos r\left\{|110\rangle\langle 111| + |111\rangle\langle 110|\right\}\Bigg].\end{aligned} \tag{16}$$

The maximum of $\langle S\rangle = \mathrm{tr}[\sigma_{ABI} S]$ was explicitly computed in Ref.[11], which has a form

$$S_{max} = 4\Bigg[\cos^2\theta_3 \cos^2 2r + 2\sin^2\theta_3\cos^2 r\Bigg]^{1/2}. \tag{17}$$

Thus, $S_{max} \geq 4$ for $a = 0$ and $S_{max} \leq 4$ for $a = \infty$.

The π-tangle for σ_{ABI} can be computed straightforwardly and its final expression is

$$\pi_{MS} = \frac{1}{3}\left[\sin^2\theta_3(2+\cos^2 r) + \sin^2 r\cos^2 r(1+\cos^2\theta_3)^2\right]. \tag{18}$$

In order to compute the three-tangle for σ_{ABI} we express σ_{ABI} in terms of eigenvectors as following:

$$\sigma_{ABI} = \Lambda_+|\Psi_+\rangle\langle\Psi_+| + \Lambda_-|\Psi_-\rangle\langle\Psi_-| \tag{19}$$

where

$$\Lambda_{\pm} = \frac{1 \pm \sqrt{\Delta}}{2} \tag{20}$$
$$|\Psi_{\pm}\rangle = \frac{1}{\mathcal{N}_{\pm}}\Big[X_{\pm}|000\rangle + Y_{\pm}|001\rangle + Z_{\pm}|110\rangle + W_{\pm}|111\rangle\Big].$$

In Eq. (20) $\Delta = \cos^2\theta_3 + \cos^2 r\left[\sin^2\theta_3 - \sin^2 r(1+\cos^2\theta_3)^2\right]$ and

$$X_{\pm} = \cos r(\mu \pm \sqrt{\Delta}) \qquad Y_{+} = Y_{-} = \sin\theta_3\cos\theta_3\sin^2 r \tag{21}$$
$$Z_{\pm} = \cos\theta_3 X_{\pm} \qquad W_{\pm} = \sin\theta_3(\cos^2 r \pm \sqrt{\Delta})$$

with $\mu = \cos^2 r - \sin^2 r\cos^2\theta_3$. The normalization constants $\mathcal{N}_{\pm}$ are

$$\begin{aligned}\mathcal{N}_{\pm}^2 &= X_{\pm}^2 + Y_{\pm}^2 + Z_{\pm}^2 + W_{\pm}^2 \\ &= \pm 2\sqrt{\Delta}\Big[(1+\mu)(\cos^2 r \pm \sqrt{\Delta}) - \sin^2 r\cos^2 r\cos^2\theta_3(1+\cos^2\theta_3)\Big].\end{aligned} \tag{22}$$

Then, it is easy to show $\langle\Psi_+|\Psi_-\rangle = 0$. Now, we define

$$|\Phi_{\pm}(\varphi)\rangle = \sqrt{\Lambda_+}|\Psi_+\rangle \pm e^{i\varphi}|\Psi_-\rangle. \tag{23}$$

Then, σ_{ABI} can be written as

$$\sigma_{ABI} = \frac{1}{2}|\Phi_+(\varphi)\rangle\langle\Phi_+(\varphi)| + \frac{1}{2}|\Phi_-(\varphi)\rangle\langle\Phi_-(\varphi)|. \tag{24}$$

The three-tangle $\tau(\Phi_{\pm})$ for $|\Phi_{\pm}(\varphi)\rangle$ are

$$\tau(\Phi_{\pm}) = 4|\tilde{X}_{\pm}\tilde{W}_{\pm} - \tilde{Y}_{\pm}\tilde{Z}_{\pm}|^2 \tag{25}$$

where $\tilde{G}_{\pm} = \sqrt{\Lambda_+}G_+/\mathcal{N}_+ \pm e^{i\varphi}\sqrt{\Lambda_-}G_-/\mathcal{N}_-$ with $G = X$, Y, Z, or W. If, thus, Eq. (24) is an optimal decomposition for σ_{ABI}, the three-tangle becomes

$$\begin{aligned}\tau(\sigma_{ABI}) = &\frac{4\Lambda_+^2}{\mathcal{N}_+^4}(X_+W_+ - Y_+Z_+)^2 + \frac{4\Lambda_-^2}{\mathcal{N}_-^4}(X_-W_- - Y_-Z_-)^2 \\ &+ \frac{4\Lambda_+\lambda_-}{\mathcal{N}_+^2\mathcal{N}_-^2}\left\{(X_+W_- + X_-W_+) - (Y_+Z_- + Y_-Z_+)\right\}^2 \\ &+ \frac{8\Lambda_+\lambda_-}{\mathcal{N}_+^2\mathcal{N}_-^2}(X_+W_+ - Y_+Z_+)(X_-W_- - Y_-Z_-)\cos 2\varphi.\end{aligned} \tag{26}$$

Since $(X_+W_+ - Y_+Z_+)(X_-W_- - Y_-Z_-) = \cos^2 r\sin^4 r\cos^4\theta_3\sin^6\theta_3 \geq 0$, we have to choose $\varphi = \pi/2$ to minimize $\tau(\sigma_{ABI})$. Then, $\tau(\sigma_{ABI})$ simply reduces to

$$\tau(\sigma_{ABI}) = \cos^2 r\sin^2\theta_3. \tag{27}$$

It is interesting to note that the three-tangle is much simpler than the π-tangle. From Eq. (17) and Eq. (27) one can derive the three-tangle-dependence of S_{max}, which is

$$S_{max} = 4\sqrt{\cos^2 2r + (5 - 4\cos^2 r - \tan^2 r)\tau(\sigma_{ABI})}. \tag{28}$$

When $a = 0$, Eq. (28) reduces to $S_{max} = 4\sqrt{1 + \tau(\sigma_{ABI})}$. Thus, the violation of the Svetlichny inequality occurs for all nonzero three-tangle. When $a = \infty$, Eq. (28) reduces to $S_{max} = 4\sqrt{2\tau(\sigma_{ABI})}$, which implies $\tau(\sigma_{ABI}) \leq 1/2$ in the infinite limit.

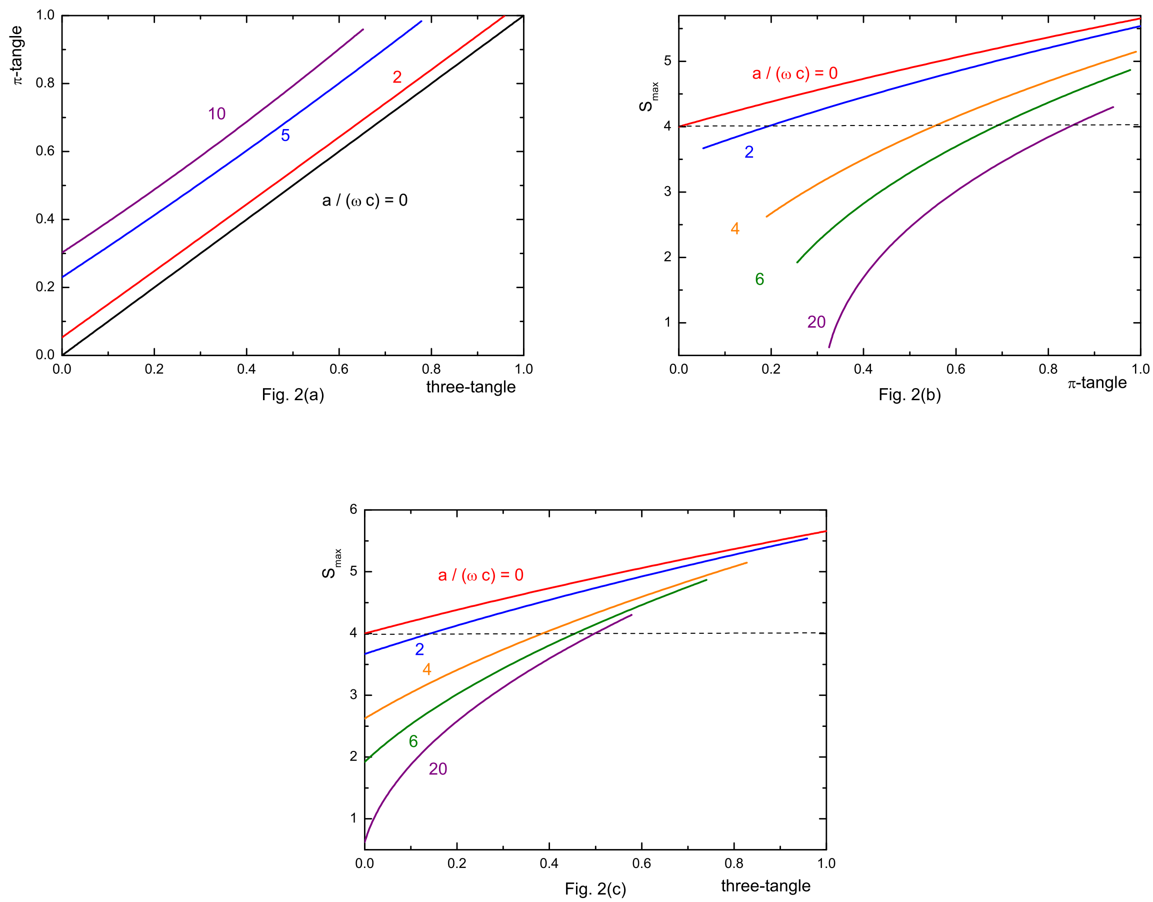

FIG. 2: (Color online) In (a) we plot the π-tangle (18) versus three-tangle (27). As Fig. 1(a) the π-tangle exhibits monotonous behavior with respect to the three-tangle. Regardless of acceleration a the π-tangle is larger than the three-tangle, which might be true generally as conjectured in Ref.[12, 23]. In (b) and (c) we plot the tripartite entanglement-dependence of S_{max}. Unlike Fig. 1(b) and Fig. 1(c) the decreasing behavior of S_{max} in small entanglement region disappears.

In Fig. 2(a) we plot the three-tangle-dependence of π-tangle for σ_{ABI} when $a = 0$, $2\omega c$, $5\omega c$, and $10\omega c$. Like Fig. 1(a) the π-tangle (18) is monotonous with respect to the three-tangle (27). Fig. 2(a) also indicates that π-tangle is in general larger than three-tangle. In Fig. 2(b) and Fig. 2(c) we plot the tripartite entanglement-dependence of S_{max}. Unlike Fig. 1(b) and Fig. 1(c) there is no decreasing behavior of S_{max} in these figures. From Fig. 2(b) and Fig. 2(c) we know that π_c and τ_c increase with increasing a if the violation of the Svetlichny inequality occurs when $\pi_{MS} > \pi_c$ and $\tau(\sigma_{ABI}) > \tau_c$. These critical values are given in Table II for various a. Table II shows that $\pi_c \to 1$ and $\tau_c \to 0.5$ in the infinite acceleration limit.

$a/\omega c$	0	2	4	6	8	10	100
π_c	0	0.191	0.250	0.685	0.746	0.780	0.901
τ_c	0	0.142	0.385	0.456	0.479	0.488	0.5

Table II: Acceleration dependence of π_c and τ_c

If Bob moves, instead of Charlie, with an uniform acceleration, the initial state $|\psi\rangle_{ABC}$ is transformed into

$$\sigma_{AIC} = \frac{1}{2}\Big[\cos^2 r|000\rangle\langle000| + \sin^2 r|010\rangle\langle010| + \cos^2\theta_3|110\rangle\langle110| + \sin^2\theta_3|111\rangle\langle111| \tag{29}$$
$$+ \cos r\cos\theta_3\{|000\rangle\langle110| + |110\rangle\langle000|\} + \cos r\sin\theta_3\{|000\rangle\langle111| + |111\rangle\langle000|\}$$
$$+ \sin\theta_3\cos\theta_3\{|110\rangle\langle111| + |111\rangle\langle110|\}\Big].$$

The maximum of $\langle S\rangle = \mathrm{tr}[\sigma_{AIC}S]$ was given in Ref. [11], which is

$$S_{max} = 4\cos r\left[\cos^2\theta_3 + 2\sin^2\theta_3\right]^{1/2}. \tag{30}$$

The π-tangle for σ_{AIC} can be straightforwardly computed and the final expression is

$$\tilde{\pi}_{MS} = \frac{1}{3}\left[1 + \sin^2\theta_3 - \cos^2 r\cos 2\theta_3 + \sin^2 r\cos 2r + \sin^2 r\sqrt{\sin^4 r + 4\cos^2 r\cos^2\theta_3}\right]. \tag{31}$$

By similar method one can compute the three-tangle for σ_{AIC}, which is exactly the same with $\tau(\sigma_{ABI})$ given in Eq. (27). Therefore, the three-tangle-dependence of S_{max} in this case is

$$S_{max} = 4\sqrt{\cos^2 r + \tau(\sigma_{AIC})}. \tag{32}$$

Eq. (32) implies that the violation of the Svetlichny inequality arises for all nonzero $\tau(\sigma_{AIC})$ when $a = 0$. It also implies that $\tau(\sigma_{AIC}) \leq 1/2$ when $a \to \infty$ limit because $S_{max} \leq 4$ in this limit.

In this paper we have examined the tripartite entanglement-dependence of $S_{max} = \max\langle S \rangle$, where S is the Svetlichny operator, when one party moves with an uniform acceleration a with respect to other parties. If the initial tripartite state is the generalized GHZ state $|\psi_g\rangle_{ABC}$, the three-tangle-dependence of S_{max} is analytically derived in Eq. (15). As Fig. 1 shows, S_{max} exhibits a decreasing behavior in the small tripartite entanglement region while it exhibits a increasing behavior in the large tripartite entanglement region. This fact seems to suggest that the tripartite entanglement is not the only physical resource for the tripartite non-locality. If initial state is the MS state $|\psi_s\rangle_{ABC}$, the explicit relations between S_{max} and three-tangle are derived in Eq. (28) and Eq. (32). In this case the decreasing behavior of S_{max} disappears as Fig. 2 shows. The a -dependence of the critical values π_*, τ_*, π_c, and τ_c is summarized in Table I and Table II.

It seems to be interesting to generalize our results to the tripartite bosonic cases[28]. In this case, however, it is highly difficult to compute S_{max} in non-inertial frame because the acceleration of one party transforms the qubit system at $a = 0$ into a qudit system for nonzero a. As Eq. (8), Eq. (17), and Eq. (30) show, the violation of the Svetlichny inequality does not occur in $a \to \infty$ limit[30] even if the tripartite entanglement does not completely vanish in this limit. This fact suggests that although there is some connection between the tripartite non-locality and the tripartite entanglement, the entanglement is not unique resource for the non-locality. Then, what are other physical resources, which are responsible for the non-locality of quantum mechanics? As far as we know, we do not have definite answer so far. We will keep on studying this issue in the future.

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